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A HYBRID ENERGY APPROACH FOR VIBRATIONAL MODELLING OF LAMINATED TRAPEZOIDAL PLATES WITH POINT SUPPORTS

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Abstract—An investigation on the free flexural vibration of symmetric angle-ply thin trapezoidal plates continuous over arbitrarily distributed point supports is reported. A hybrid energy approach which combines the *pb*-2 Rayleigh–Ritz method with the Lagrangian multiplier method is proposed for the modelling of the aforementioned plate problem. The *pb*-2 Rayleigh–Ritz method uses a set of Ritz functions generated from the product of a two-dimensional polynomial and the equations of boundaries each raised to the power of 0, 1 or 2 corresponding to a free, simply-supported or clamped edge, respectively. The geometric boundary conditions associated with the point supports are satisfied through the use of Lagrangian multipliers. In this paper, some new solutions for the natural frequencies of several laminated trapezoidal plates with different stacking sequences and location of point supports are presented. The first known mode shapes by means of contour plots for such laminated plates are also included.

NOTATION

a, c	lengths of two parallel sides of trapezoidal plate (see Fig. 1)
b	height of trapezoidal plate
C_i	coefficients
$[D]$	bending stiffness coefficients
D_0	$E_1h^3/12(1-\nu_{12}\nu_{21})$
E_1, E_2	Young's moduli parallel to and perpendicular to fibres
h	plate thickness
$[K]$	bending curvatures
$[M]$	moment resultants
p	degree set of polynomial space
R	plate domain
T	maximum kinetic energy
V	maximum strain energy
$W(\xi, \eta)$	displacement function
x, y	Cartesian coordinates
β	fibre orientation angle
$\Phi_i(\xi, \eta)$	basic function
η	y/b
λ	frequency parameter $(\rho h\omega^2 a^4/D_0)$
ρ	density per unit area of plate
ν_{12}, ν_{21}	Poisson's ratios
ω	natural radian frequency
ξ	x/a.

1. INTRODUCTION

Only limited research work has been reported in the open literature concerning the free flexural vibrations of thin symmetrically laminated trapezoidal plates. However the practical applications of such laminates in aircraft and aerospace industries are very important because lighter and stiffer structures may be built as the composite materials are utilized.

For static and dynamic analyses, the analytical or exact solutions to the symmetric angle-ply thin plates are difficult (and perhaps impossible) due to the presence of odd derivatives in the governing differential equation of motion. In the open literature, only a few exact vibration solutions for laminated cross-ply simply-supported rectangular plates are available (Whitney and Leissa, 1969; Lin and King, 1974; Baharlou and Leissa, 1987).

A recent publication by Leissa and Narita (1989) has presented a set of comprehensive approximate Ritz-vibration solutions for symmetrically laminated thin simply-supported rectangular plates. Following that Chow *et al.* (1992) have proposed a set of two-dimensional orthogonal polynomial functions (Liew, 1990; Liew and Lam, 1991) which is used as the admissible displacement function in the Rayleigh–Ritz method to study the vibration of the symmetric angle-ply rectangular plates with any possible combination of classical edge support conditions. Some numerical results for the natural frequencies and mode shapes of such plates were presented.

For vibration of symmetric angle-ply thin plates of other shapes, only a few papers can be found in the open literature. Nair and Durvasula (1974) have presented a formulation based on the orthotropic plate theory with arbitrary orientation of principal axes of orthotropy. Natural frequencies for several single-layer composite skew plates were presented. Some numerical results for a fully clamped single-layer skew plate subjected to in-plane forces have been published by Srinivasan and Ramachandran (1975). Recently Liew (1992) has presented a study on the free vibration of symmetric angle-ply trapezoidal plates using a set of two-dimensional orthogonal polynomials in the Rayleigh–Ritz method. Natural frequencies and mode shapes for several cantilever trapezoidal plates were obtained. The method was further extended to study the same problem but with internal elastic point constraints by modifying the general energy functional with an additional term which associated with the elastic springs (Liew and Lam, 1992). An investigation into the effects of elastic spring constants on the natural frequencies and mode shapes of symmetrically laminated trapezoidal plates was carried out.

This paper extends the earlier work by the author (Liew, 1992) to study the free vibration analysis of symmetric angle-ply trapezoidal plates with point supports arbitrarily distributed inside the plate domain or along the edges. The present study deals with plates on simple point supports (deflection, $w = 0$) which is different from the work by Liew and Lam (1992) who considered the plates supported by elastic points. Of course, by considering the elastic stiffness to be large, it leads to the same solutions as the present approach. One should note that as the elastic points increase, unstable solutions may be encountered due to the large stiffness resulting from the large spring constants assumed. This setback will not be encountered in the hybrid energy approach.

The present analysis is performed using a hybrid *pb*-2 Rayleigh–Ritz–Lagrangian multiplier approach. In the Rayleigh–Ritz method, the admissible displacement function (Liew and Wang, 1992) employed is a set of *pb*-2 Ritz functions which consists of the product of a two-dimensional polynomial (*p*-2) and a basic function (*b*). The basic function is the product of the equations of the piecewise continuous boundary shape each raised to the power of 0, 1 or 2, corresponding to a free, simply-supported or clamped edge, respectively. The set of functions generated automatically satisfies the geometric boundary conditions of the plate at the outset. For the geometric boundary conditions of zero deflections ($w = 0$) associated with the point supports are satisfied by introducing the Lagrangian multipliers.

Several trapezoidal plates with different numbers of layers, stacking sequences and point locations are studied. The examples considered by Liew and Lam (1992) are also solved to serve as the purpose of comparison. Several new problems are introduced. The first few natural frequency parameters and mode shapes for these new trapezoidal plates are reported herein.

2. PROBLEM DEFINITION

Consider a thin, fibre-reinforced composite, laminated angle-ply, points supported trapezoidal plate lying in the xy-plane, and bounded by $-a/2 \leqslant x \leqslant a/2$ and $-b/2 \leqslant y \leqslant b/2$, as shown in Fig. 1. The plate, with thickness h in the z-direction, consists of n layers of orthotropic plies perfectly bonded together by a matrix material. The reference plane $z = 0$ is considered to be located at the undeformed middle plane as shown in Fig. 2. The fibre direction within a layer is indicated by the angle β. The modulus of elasticity for the layer parallel to the fibres is denoted by E_1 and perpendicular to the fibres by E_2.

In the present analysis, only plates with stacking layers symmetric about the mid-plane are considered. By this special symmetrical arrangement, the coupling between the transverse

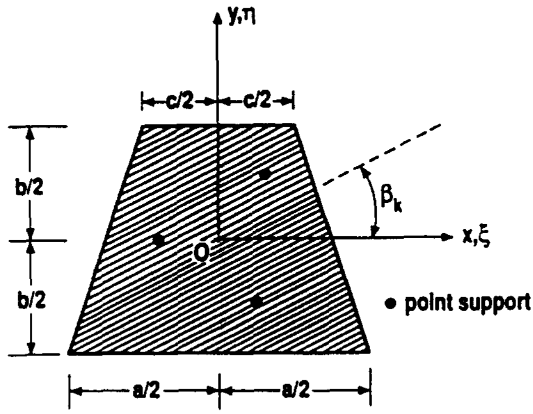

Fig. 1. Geometry of a trapezoidal plate with fibre direction β_k continuous over point supports.

bending and in-plane stretching is avoided. The problem is to determine the natural frequencies and mode shapes for the symmetric angle-ply trapezoidal plate with arbitrarily distributed point supports.

3. METHOD OF SOLUTION

An attempt to solve the problems is made by using the Rayleigh–Ritz approach with a set of *pb-2* Ritz functions (referred to as the *pb-2* Rayleigh–Ritz method) together with the Lagrangian multiplier method.

The strain energy for the plate due to bending can be expressed as

$$V = \frac{1}{2}\int\int_R [M][K]\,\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

where the integration is carried out over the entire plate domain R and

$$[M] = [M_x, M_y, M_{xy}]^{\mathrm{T}}, \tag{2}$$

$$[K] = [K_x, K_y, K_{xy}], \tag{3}$$

in which $[M]$ is the moment resultant and $[K]$ is the bending curvature.

The bending curvatures are related to the displacements by

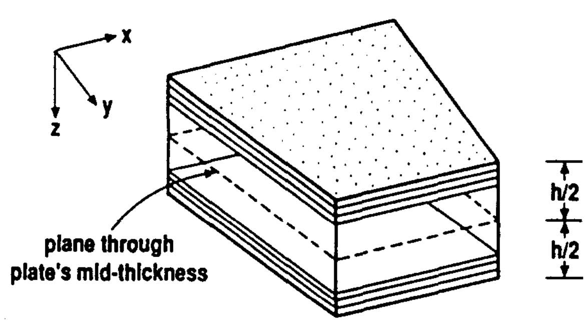

Fig. 2. Layer coordinates and orientation for laminates.

$$K_x = \frac{\partial^2 W}{\partial x^2}, \tag{4}$$

$$K_y = \frac{\partial^2 W}{\partial y^2}, \tag{5}$$

$$K_{xy} = 2\frac{\partial^2 W}{\partial x\,\partial y}. \tag{6}$$

For anisotropic materials, the moment resultants are given by

$$[M] = [D][K], \tag{7}$$

where $[D]$ is a 3×3 symmetric matrix of bending stiffness coefficients.

For symmetric angle-ply laminates, the coefficients of the bending stiffness matrix are given by

$$D_{ct} = \frac{1}{3}\sum_{k=1}^{m} (N_{ct})_k (h_k^3 - h_{k-1}^3); \quad c, t = 1, 2, 6, \tag{8}$$

where $(N_{ct})_k$ is the reduced stiffness of the kth ply which is defined by the elastic constants of the layer and fibre orientation angle β_k. The reduced stiffness for the kth ply $(N_{ct})_k$ can be expressed as

$$N_{11_k} = Q_{11_k}\cos^4\beta_k + 2(Q_{12_k} + 2Q_{66_k})\sin^2\beta_k\cos^2\beta_k + Q_{22_k}\sin^4\beta_k, \tag{9}$$

$$N_{12_k} = (Q_{11_k} + Q_{22_k} - 4Q_{66_k})\sin^2\beta_k\cos^2\beta_k + Q_{12_k}(\sin^4\beta_k + \cos^4\beta_k), \tag{10}$$

$$N_{22_k} = Q_{11_k}\sin^4\beta_k + 2(Q_{12_k} + 2Q_{66_k})\sin^2\beta_k\cos^2\beta_k + Q_{22_k}\cos^4\beta_k, \tag{11}$$

$$N_{16_k} = (Q_{11_k} - Q_{12_k} - 2Q_{66_k})\sin\beta_k\cos^3\beta_k + (Q_{12_k} - Q_{22_k} + 2Q_{66_k})\sin^3\beta_k\cos\beta_k, \tag{12}$$

$$N_{26_k} = (Q_{11_k} - Q_{12_k} - 2Q_{66_k})\sin^3\beta_k\cos\beta_k + (Q_{12_k} - Q_{22_k} + 2Q_{66_k})\sin\beta_k\cos^3\beta_k, \tag{13}$$

$$N_{66_k} = (Q_{11_k} + Q_{22_k} - 2Q_{12_k} - 2Q_{66_k})\sin^2\beta_k\cos^2\beta_k + Q_{66_k}(\sin^4\beta_k + \cos^4\beta_k), \tag{14}$$

where

$$Q_{11_k} = \frac{E_{1_k}}{1 - \nu_{12_k}\nu_{21_k}}, \tag{15}$$

$$Q_{12_k} = \frac{\nu_{12_k}E_{2_k}}{1 - \nu_{12_k}\nu_{21_k}}, \tag{16}$$

$$Q_{22_k} = \frac{E_{2_k}}{1 - \nu_{12_k}\nu_{21_k}}, \tag{17}$$

$$Q_{66_k} = G_{12_k}, \tag{18}$$

$$\nu_{21_k}E_{1_k} = \nu_{12_k}E_{2_k}, \tag{19}$$

in which E_{1k} and E_{2k} are the Young's moduli parallel to and perpendicular to the fibres and ν_{12k} and ν_{21k} are the corresponding Poisson's ratios.

Substituting eqns (2)–(8) into eqn (1), the strain energy becomes

$$V = \frac{1}{2}\int\!\!\int_R \left\{ D_{11}\left[\frac{\partial^2 W}{\partial x^2}\right]^2 + 2D_{12}\left[\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}\right] + D_{22}\left[\frac{\partial^2 W}{\partial y^2}\right]^2 + 4D_{16}\left[\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial x\,\partial y}\right] \right.$$
$$\left. + 4D_{26}\left[\frac{\partial^2 W}{\partial y^2}\frac{\partial^2 W}{\partial x\,\partial y}\right] + 4D_{66}\left[\frac{\partial^2 W}{\partial x\,\partial y}\right]^2 \right\} \mathrm{d}x\,\mathrm{d}y. \quad (20)$$

The maximum kinetic energy of the plate during small amplitude vibration is given by

$$T = \tfrac{1}{2}\rho h \omega^2 \int\!\!\int_R W^2(x, y)\,\mathrm{d}x\,\mathrm{d}y, \quad (21)$$

where ρ is the mass per unit area of plate, h is the thickness and ω is the angular frequency of vibration.

The total energy functional for the plate can now be written as

$$F = V - T. \quad (22)$$

The displacement function $W(\xi, \eta)$ may be expressed as

$$W(\xi, \eta) = \sum_{q=0}^{p} \sum_{i=0}^{q} C_r \Phi_r(\xi, \eta), \quad (23)$$

where $\xi = x/a$, $\eta = y/b$, p is the degree set of polynomial space, C_r are the unknown coefficients and

$$r = \frac{(q+1)(q+2)}{2} - i, \quad (24)$$

$$\Phi_r(\xi, \eta) = (\xi^i \eta^{q-i})\Phi_1(\xi, \eta). \quad (25)$$

The basic function $\Phi_1(\xi, \eta)$ is defined as

$$\Phi_1(\xi, \eta) = \prod_{i=1}^{4} [\Gamma_i(\xi, \eta)]^{\Omega_i} \quad (26)$$

in which Γ_i is the boundary equation of the ith supporting edge, and Ω_i takes on 0, 1 or 2 corresponding to a free, simply-supported or clamped edge, respectively.

For the cantilever trapezoidal plate considered here, the basic function is simply given by

$$\Phi_1(\xi, \eta) = (\eta + \tfrac{1}{2})^2. \quad (27)$$

If the plate has N point supports either along its edges or internally, the deflection surface has to be constrained as

$$W_i = 0; \quad i = 1, 2, 3, \ldots, N, \quad (28)$$

where W_i is the deflection at the ith point support. The constraints may be satisfied by augmenting the functional, F, of eqn (22) to

$$F^* = V - T + \sum_{i=1}^{N} \Lambda_i W_i(\xi_i, \eta_i), \tag{29}$$

where Λ_i are the Lagrangian multipliers and (ξ_i, η_i) are the position coordinates of the point supports.

The minimization of the augmented functional, F^*, with respect to C_i and Λ_i leads to the governing eigenvalue equation

$$\left(\begin{bmatrix} K & L^T \\ L & 0 \end{bmatrix} - \lambda \begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix}\right) \begin{Bmatrix} C \\ \Lambda \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{30}$$

where $\{C\} = \{C_1, C_2, \ldots, C_n\}^T$, $\{\Lambda\} = \{\Lambda_1, \Lambda_2, \ldots, \Lambda_N\}^T$, and the elements in the matrices are

$$K_{ij} = \frac{1}{D_0}\{D_{11}R_{ij}^{2020} + D_{22}R_{ij}^{0202} + D_{12}(R_{ij}^{0220} + R_{ij}^{2002})$$

$$+ 2D_{16}(R_{ij}^{2011} + R_{ij}^{1120}) + 2D_{26}(R_{ij}^{0211} + R_{ij}^{1120}) + 4D_{66}R_{ij}^{1111}\}, \tag{31}$$

$$\lambda = \frac{\rho h \omega^2 a^4}{D_0}, \quad D_0 = \frac{E_1 h^3}{12(1 - \nu_{12}\nu_{21})}, \tag{32}$$

$$L_{qi} = \Phi_i(\xi_q, \eta_q), \quad q = 1, 2, \ldots, N, \tag{33}$$

$$M_{ij} = R_{ij}^{0000} \tag{34}$$

and

$$R_{ij}^{rsvw} = \iint_R \left[\frac{\partial^{r+s}\Phi_i(\xi, \eta)}{\partial \xi^r \partial \eta^s}\right]\left[\frac{\partial^{v+w}\Phi_j(\xi, \eta)}{\partial \xi^v \partial \eta^w}\right] d\xi \, d\eta, \tag{35}$$

in which $i, j = 1, 2, \ldots, m$ and m is the number of polynomial terms in a pth degree set which is equal to $(p+1)(p+2)/2$.

For vibration analysis, the eigenvalues are obtained by solving the set of homogeneous equations (30). Back-substitution yields the coefficient vectors; substitution of these coefficient vectors into eqn (23) gives the mode shapes of the plate.

4. NUMERICAL EXAMPLES AND DISCUSSION

Several examples have been selected to demonstrate the applicability and accuracy of the proposed method. In this paper, symmetric angle-ply trapezoidal plates with different stacking sequences, angle of fibre orientations, number of layers and location of point supports are considered. The eigenvalues obtained for the examples are expressed in terms of the non-dimensional frequency parameter $(\rho h \omega^2 a^4 / D_0)^{1/2}$. In order to compare the results published by Liew and Lam (1992), the same graphite/epoxy (G/E) composite is chosen. The detailed material properties of graphite/epoxy (G/E) composite are given in Table 1.

Table 1. Material properties of graphite/epoxy (G/E) composite

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
G/E	138	8.96	7.1	0.30

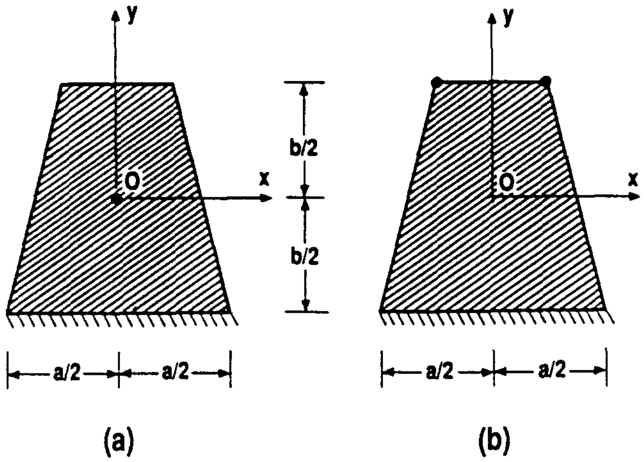

Fig. 3. Boundary conditions for the point supported trapezoidal plates under investigation.

4.1. *Centrally-located point supports*

The first set of problems considered is a cantilever, laminated trapezoidal plate ($a/b = 1$) with the edge clamped at $y = -1/2$ which is continuous over a centrally-located point support ($x = 0$, $y = 0$) as shown in Fig. 3(a).

Initially, the method is applied to solve the same problems as presented previously by Liew and Lam (1992) which are the eight-ply laminated plate with stacking sequence of $[(0°, 90°, 90°, 0°)]_{sym}$ and 16-ply with stacking sequence of $[(0°, 45°, -45°, 90°)_2]_{sym}$. By using the degree set of $p = 10$ in the deflection function, the results are obtained and tabulated in Table 2 together with the solutions of Liew and Lam (1992). It can be seen that the present results and those values of Liew and Lam are in close agreement.

Two new examples are included in this section. The convergence patterns of the frequency parameters for (a) an eight-ply $[22.5°, -22.5°, 67.5°, -67.5°]_{sym}$, and (b) a 16-ply $[(30°, -30°, 60°, -60°)_2]_{sym}$ laminated trapezoidal plates ($a/b = 1$, $c/a = 2/5$) are given in Table 3. It can be seen that, for both cases, convergent results are obtained when $p = 10$. The displacement contour plots of the first six mode shapes for the eight-ply and 16-ply laminated trapezoidal plates with $c/a = 2/5$ and $4/5$ are presented in Figs 4 and 5, respectively.

4.2. *Point supports located on the edge*

The second set of problems considered is also a cantilever, laminated trapezoidal plate ($a/b = 1$) with the edge clamped at $y = -1/2$ [see Fig. 3(b)] but now with two point supports located (a) for $c/a = 2/5$ at $x = -1/5$, $y = 1/2$ and at $x = 1/5$, $y = 1/2$, and (b) for $c/a = 4/5$ at $x = -2/5$, $y = 1/2$ and $x = 2/5$, $y = 1/2$, respectively.

Table 2. Comparison of frequency parameters $(\rho h\omega^2 a^4/D_0)^{1/2}$ for symmetric angle-ply trapezoidal plates with a centrally-located point support ($a/b = 1$)

c/a	Source	Mode sequence number 1	2	3	4	5	6
		(a) $[(0°, 90°, 90°, 0°)]_{sym}$					
2/5	Liew and Lam	8.49	8.67	26.58	27.81	48.79	56.33
	Present	8.49	8.70	26.61	27.81	48.83	56.36
4/5	Liew and Lam	5.18	6.69	19.79	20.94	30.04	48.88
	Present	5.18	6.69	19.80	20.96	30.11	48.96
		(b) $[(0°, 45°, -45°, 90°)_2]_{sym}$					
2/5	Liew and Lam	8.67	11.28	30.61	31.86	45.74	61.30
	Present	8.67	11.28	30.65	31.90	45.81	61.48
4/5	Liew and Lam	6.90	7.49	21.28	23.92	42.95	45.21
	Present	6.90	7.51	21.33	24.01	42.97	45.32

Table 3. Convergence patterns of frequency parameters $(\rho h\omega^2 a^4/D_0)^{1/2}$ for symmetric angle-ply trapezoidal plates with a centrally-located point support ($a/b = 1$, $c/a = 2/5$)

Degree p	Mode sequence number 1	2	3	4	5	6
	(a) $[(22.5°, -22.5°, 67.5°, -67.5°)]_{sym}$					
7	9.10	11.76	30.43	32.91	49.27	61.76
8	9.05	11.75	29.68	32.50	49.15	61.31
9	9.04	11.75	29.62	32.48	49.07	61.14
10	9.03	11.75	29.60	32.48	49.06	61.12
	(b) $[(30°, -30°, 60°, -60°)_2]_{sym}$					
7	7.87	12.13	31.88	33.15	41.26	61.28
8	7.84	12.13	31.18	32.81	41.24	61.19
9	7.83	12.12	31.08	32.78	41.19	61.17
10	7.83	12.12	31.07	32.78	41.19	61.16

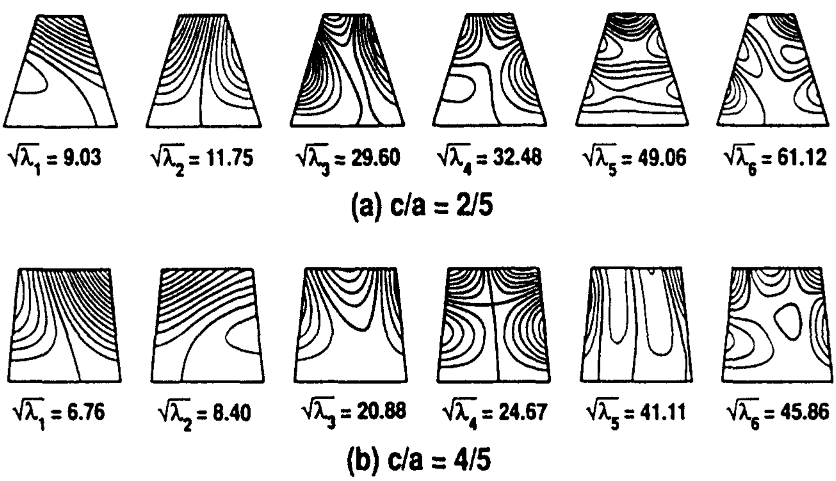

Fig. 4. Contour plots for the mode shapes of the eight-ply G/E centrally point supported ($x = 0$, $y = 0$) laminated trapezoidal plate ($a/b = 1$) with stacking sequence of $[22.5°, -22.5°, 67.5°, -67.5°]_{sym}$.

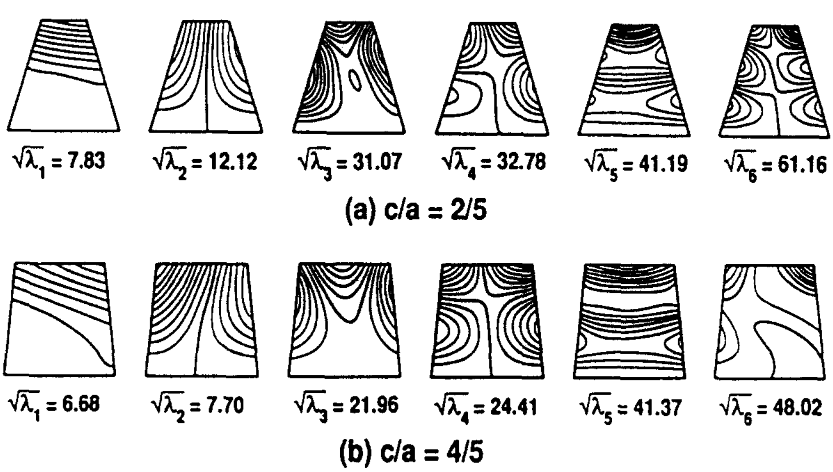

Fig. 5. Contour plots for the mode shapes of the 16-ply G/E centrally point supported ($x = 0$, $y = 0$) laminated trapezoidal plate ($a/b = 1$) with stacking sequence of $[(30°, -30°, 60°, -60°)_2]_{sym}$.

Table 4. Comparison of frequency parameters $(\rho h\omega^2 a^4/D_0)^{1/2}$ for symmetric angle-ply trapezoidal plates with two point supports located at $x = -1/5$, $y = 1/2$ and at $x = 1/5$, $y = 1/2$ $(a/b = 1)$

c/a	Source	Mode sequence number 1	2	3	4	5	6
		(a) $[(0°, 90°, 90°, 0°)]_{sym}$					
2/5	Liew and Lam	12.19	15.78	33.27	40.16	42.56	53.00
	Present	12.20	15.83	33.28	40.18	42.64	53.11
4/5	Liew and Lam	8.84	13.69	18.59	27.15	35.92	44.27
	Present	8.84	13.70	18.63	27.23	36.01	44.30
		(b) $[(0°, 45°, -45°, 90°)_2]_{sym}$					
2/5	Liew and Lam	11.73	18.37	33.05	45.43	45.85	56.36
	Present	11.73	18.40	33.07	45.43	45.91	56.41
4/5	Liew and Lam	9.48	15.48	18.01	36.29	38.17	47.68
	Present	9.48	15.48	18.04	36.37	38.23	47.72

Similarly, two known examples (Liew and Lam, 1992) are solved to verify the accuracy of the present approach. There are the laminated plates with the stacking of: (a) $[(0°, 90°, 90°, 0°)]_{sym}$ and (b) $[(0°, 45°, -45°, 90°)_2]_{sym}$. The results obtained using $p = 10$ and the published values are given in Table 4. It is evident that the comparison is shown in good agreement.

Again, two new examples are considered here. Table 5 shows the convergence of the frequency parameters for (a) an eight-ply $[22.5°, -22.5°, 67.5°, -67.5°]_{sym}$, and (b) a 16-ply $[(30°, -30°, 60°, -60°)_2]_{sym}$ laminated trapezoidal plates ($a/b = 1$, $c/a = 2/5$). Similarly convergent solutions for the first six modes can be obtained when $p = 10$. The mode shapes for the two laminated trapezoidal plates with $c/a = 2/5$ and $4/5$ are presented in Figs 6 and 7 together with the relevant frequency parameters given below each mode shape.

5. CONCLUSIONS

The paper presents some new vibration solutions for symmetric angle-ply trapezoidal plates continuous over point supports of arbitrary distribution. A hybrid *pb-2* Rayleigh–Ritz–Lagrangian multiplier method was proposed to solve the aforementioned plate problems. The method has been shown to give accurate frequency parameters and mode shapes for these plates through the comparisons and convergence tests.

From the present study, it is concluded that different numbers of layers and combinations of fibre orientations may affect the vibratory response for the plate problems considered.

It should be remarked that the proposed method is capable of analysing symmetric angle-ply plates or arbitrary shape and any combinations of boundary conditions. It can

Table 5. Convergence patterns of frequency parameters $(\rho h\omega^2 a^4/D_0)^{1/2}$ for symmetric angle-ply trapezoidal plates with two point supports located at $x = -1/5$, $y = 1/2$ and at $x = 1/5$, $y = 1/2$ $(a/b = 1$, $c/a = 2/5)$

Degree p	Mode sequence number 1	2	3	4	5	6
	(a) $[(22.5°, -22.5°, 67.5°, -67.5°)]_{sym}$					
7	12.67	18.68	32.52	45.83	47.21	60.83
8	12.67	18.67	32.25	45.43	46.22	57.88
9	12.66	18.67	32.23	45.35	46.20	57.57
10	12.66	18.67	32.22	45.35	46.18	57.55
	(b) $[(30°, -30°, 60°, -60°)_2]_{sym}$					
7	10.52	19.00	31.66	45.06	45.81	62.04
8	10.51	19.00	31.60	44.42	45.51	59.69
9	10.51	19.00	31.56	44.38	45.49	59.35
10	10.50	19.00	31.56	44.38	45.49	59.35

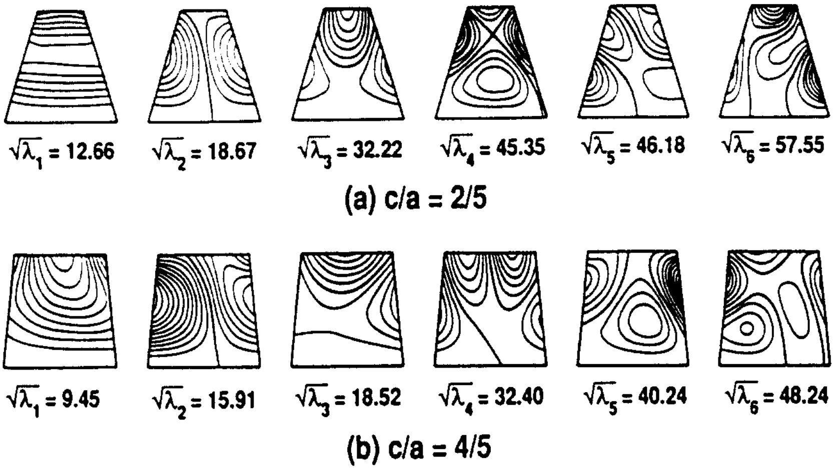

Fig. 6. Contour plots for the mode shapes of eight-ply G/E laminated trapezoidal plate ($a/b = 1$) having two point supports located at: (a) $x = -1/5$, $y = 1/2$ and $x = 1/5$, $y = 1/2$ ($c/a = 2/5$), and (b) $x = -2/5$, $y = 1/2$ and $x = 2/5$, $y = 1/2$ ($c/a = 4/5$) with stacking sequence of $[22.5^\circ, -22.5^\circ, 67.5^\circ, -67.5^\circ]_{sym}$.

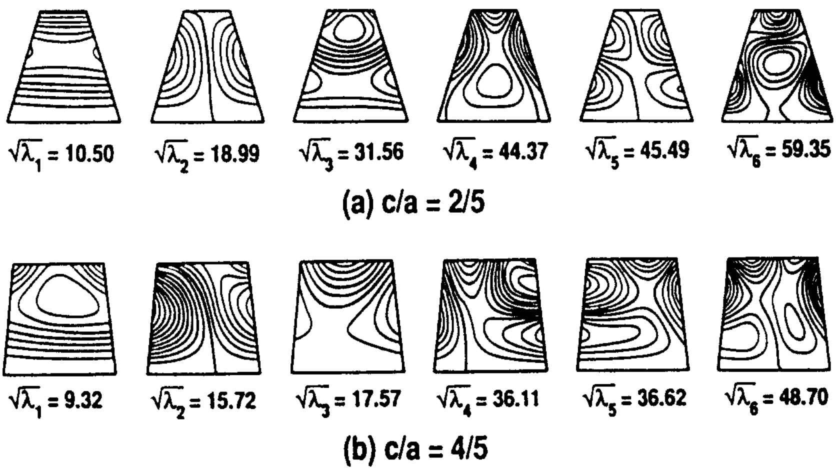

Fig. 7. Contour plots for the mode shapes of 16-ply G/E laminated trapezoidal plate ($a/b = 1$) having two point supports located at: (a) $x = -1/5$, $y = 1/2$ and $x = 1/5$, $y = 1/2$ ($c/a = 2/5$), and (b) $x = -2/5$, $y = 1/2$ and $x = 2/5$, $y = 1/2$ ($c/a = 4/5$) with stacking sequence of $[(30^\circ, -30^\circ, 60^\circ, -60^\circ)_2]_{sym}$.

also be employed to perform the bending and buckling analyses of symmetric angle-ply plates with the appropriate energy functionals.

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